PROPOSITIONALISM*

"Every proposition is the result of truth-operations on elementary propositions," declares Wittgenstein in the *Tractatus*.[1] The analysis of propositions leads to these elementary ones,[2] which can neither contradict one another[3] nor be deduced from one another.[4] As Bell and Demopoulos note,[5] elementary propositions in the *Tractatus* may thus be regarded as free generators in the logician's sense. They are logically independent, cannot be further analyzed, and generate all other propositions.

By what means do elementary propositions generate all other propositions? We are told in 5.32 that "all truth-functions are results of successive applications to elementary propositions of a finite number of truth-operations." This suggests the bold hypothesis of *propositionalism* (my label).[6] According to it, logical consequence is capturable by propositional logic, a logic whose formulas are finite truth-functional compounds of sentence letters. *All* logical relations, propositionalists maintain, can be accurately reflected by a propositional formalization.

Now the *Tractatus* makes for an attractive incipit, but it is only a very loose historical inspiration for my paper and will not be its focus. So I want to set aside the question of how faithful propositionalism is to the Tractarian vision. (Clearly, not *that* faithful, seeing as Wittgenstein countenanced existential and universal quantification and free variables, identified elementary propositions with concatenations of names rather than sentence letters, and so on.) My interest, rather,

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[1] Ludwig Wittgenstein, *Tractatus Logico-Philosophicus* (New York: Routledge, 1921), 5.3.

[2] *Ibid.*, 4.221.

[3] *Ibid.*, 4.211.

[4] *Ibid.*, 5.134.

[5] John Bell and William Demopoulos, "Elementary Propositions and Independence," *Notre Dame Journal of Formal Logic*, XXXVII, 1 (1996): 112–24.

[6] Not to be confused with propositionalism in philosophy of language, as in Graeme Forbes, *Attitude Problems: An Essay on Linguistic Intensionality* (Oxford: Oxford University Press, 2006), chapter 4. An unrelated view, this other form of propositionalism has it that the logical forms of sentences containing intensional intransitive verbs involve sentential complements.

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is in propositionalism and its alleged shortcomings. What I want to know is whether propositional logic captures the logical structure of language; and if not, to understand why not.

To virtually all contemporary logicians, the first question has an easy answer. *Of course* propositional logic is incapable of capturing the logical structure of language. Its deficiencies, known even to novice logicians, are what motivate the move to predicate/first-order logic. Propositional logic is too weak to capture the validity of some evidently valid arguments, which, however, can be validly formalized in predicate/first-order logic and various extensions. The two arguments in the table's left-hand column illustrate the point.

Argument	Propositional Formalization	First-Order Formalization
Bruin is a bear ∴ There is a bear	p $\therefore q$	Bb $\therefore \exists x Bx$
Zeno is a tortoise All tortoises are toothless ∴ Zeno is toothless	p q $\therefore r$	To $\forall x(Tx \to Hx)$ $\therefore Ho$

Figure 1.

The "Bruin" and "Zeno" arguments are both valid; their propositional formalizations are invalid; and their first-order formalizations are valid.[7] According to a recent textbook writer, the Zeno argument is valid but not propositionally valid, so "[i]n order to capture the validity of arguments like this one about Zeno, a formal language more powerful and more sophisticated than the language $\mathcal{L}_1$ of propositional logic is required."[8] Halbach here expresses a universal attitude among logicians, and quotes to a similar effect can be multiplied to your heart's content. The moral of the Bruin and Zeno examples, borne out by countless similar ones, is that the validity of some natural-language arguments turns on features that go beyond propositional logic. Propositionalism is dead in the water.

A further and more general moral is usually drawn from such examples. To capture the validity of an argument, one often needs to

[7] We assume standard semantics for propositional logic and for first-order logic (understood to include identity). The notion of natural-language validity I incline toward is that of necessary truth-preservation as a matter of form. But the morals that follow hold on many other understandings. As will be clear from sections III–IV, propositionalism is even easier to uphold on a non-formal account of consequence, since the grammatical constraint introduced below no longer applies.

[8] Volker Halbach, *The Logic Manual* (Oxford: Oxford University Press, 2010), p. 74.

move from a weaker logic to a stronger one. The classic example remains that of propositional versus first-order logic, but there are others. As against first-orderism—which sees first-order logic as sufficient and necessary for capturing the logical structure of language—many logicians have, for instance, argued that there are arguments whose validity can only be captured in stronger logics such as modal logic, second-order logic, infinitary logics, and so on. Much of the story of philosophical logic of the past several decades can be seen as the unfolding of this very moral.

So far, so very familiar. But wait. In illustrating this apparently familiar shortcoming of propositional logic, we smuggled in an assumption. In the Bruin argument in Figure 1, we assumed that the formalization of the premise is p and that of the conclusion q; and in the Zeno argument, we formalized the premises as p and q and the conclusion as r. What if we proceed differently? What if, for example, our formalization deployed complex propositional formulas?

I. DIFFERENT FORMALIZATIONS

Propositionalism *per se* is not committed to any particular formalization into propositional logic. So consider what would happen if we employed the following propositional formalizations instead:

Argument	Propositional Formalization
Bruin is a bear ∴ There is a bear	p $\therefore p \vee q$
Zeno is a tortoise All tortoises are toothless ∴ Zeno is toothless	p $(p \to q) \wedge (r \to s)$ $\therefore q$

Figure 2.

We can think of q in the Bruin argument as formalizing the sentence “There is a bear other than Bruin,” so that $p \vee q$ corresponds to the formalization of “There is a bear.” And in the Zeno argument, $r \to s$ formalizes the proposition that any tortoise other than Zeno is toothless. With these formalizations, the arguments are now rendered valid. And with their intended interpretations, the propositional sentences are truth-conditionally equivalent to their natural-language counterparts. So it looks like we might have been too swift in rejecting propositionalism. For propositional logic, it is now apparent, *can* respect the two arguments’ validity. More precisely, there is a way to formalize the arguments in propositional logic such that the resulting formalized arguments are propositionally valid.

At this point, most logicians will wish to protest. The arguments' propositional validity was assured by cooking up an unnatural formalization. The first argument's conclusion, "There is a bear," is, they will say, propositionally atomic, and so must be formalized as a propositional atom such as q (its formalization in Figure 1) rather than as a disjunction (its formalization in Figure 2). And although we can come up with unnatural formalizations that do the job in these toy cases, it is surely too much to expect propositional logic to match first-order logic across the board.

Let us clearly separate these two objections. The first is that the Figure 2 formalizations are "unnatural." What is meant by this is that the propositional formalizations do not respect the English sentences' grammar. The first-order formalizations in Figure 1, in contrast, cleave much more closely to the grammar of the original English. And formalization is a task that is constrained to respect grammar as well as to mirror implication relations.[9] The second objection is that there is no guarantee that the trick in Figure 2 can be applied more generally. For the simple Bruin and Zeno arguments, one can come up with crafty, if unnatural, formalizations that capture the original arguments' validity. But is there a propositional formalization—even an unnatural one—that will work for *all* arguments?

In this article, a sequel to "Capturing Consequence,"[10] my aim is to address both of these objections, concentrating on the dispute between propositionalism and first-orderism. In section II, I offer a brief summary of my earlier article, which spoke to the second objection. I showed there that for any given first-order formalization of the totality of English sentences, there is a propositional one that is just as good at capturing facts about English validity and invalidity. Informally put (an exact formulation will follow in section II): propositional logic can match first-order logic implicationally. Section III explicates the grammatical criterion. In section IV, I extend the section II result to show that for any given first-order formalization, there is a propositional one that not only matches first-order logic

[9] Some well-known formalization projects such as Russell's theory of descriptions and Davidson's event analysis of action sentences appear to fly in the face of grammar; see Bertrand Russell, "On Denoting," *Mind*, XIV, 56 (1905): 479–93; and Donald Davidson, "The Logical Form of Action Sentences," in Nicholas Rescher, ed., *The Logic of Decision and Action* (Pittsburgh: University of Pittsburgh Press, 1967), pp. 81–95, reprinted with criticism, comment, and defense in his *Essays on Actions and Events* (Oxford: Oxford University Press, 2001), pp. 105–48. For such authors, grammar plays *some* role, but a secondary one. As we shall see, the more grammar's role is played down, the more grist is added to the propositionalist's mill.

[10] A. C. Paseau, "Capturing Consequence," *The Review of Symbolic Logic*, XII (2019): 271–95.

implicationally but also, by the propositional logician's lights, meets the grammatical constraint. Thus propositional logic can match first-order logic both implicationally and grammatically (exact formulation to follow in section IV), as the propositional logician sees it. That is, propositional formalizations can match first-order ones in terms of which arguments they formalize as valid (respectively: invalid) *and*, at the same time, be just as faithful to the English sentences' grammar, as the propositional logician construes it. Section V sets out the real reasons for preferring first-order to propositional logic as a means of capturing logical relations. Section VI briefly concludes.

So why is first-order logic superior to propositional logic? To this question, virtually all logicians trot out the answer given at the start of this article. But as I hope this article will demonstrate, this pat answer is wrong. The correct one, offered in section V, is much more interesting.

II. MATCHING CONSEQUENCE

Let PL_ω be countable propositional logic. That is, PL_ω has a countable infinity of atoms/sentence letters ($p_1, \ldots, p_n, \ldots$), a countable and expressively adequate set of truth-functional connectives such as $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$, the usual formation rules, and its standard consequence relation $\vDash_{\mathsf{PL}_\omega}$. $\mathit{Sen}(\mathsf{PL}_\omega)$ is PL_ω's set of sentences. This is the logic the propositionalist sees as adequately capturing the logical structure of language.

Similarly, let FOL_ω be countable first-order logic with identity. That is, FOL_ω is first-order logic with a countable infinity of variables ($x_1, \ldots, x_n, \ldots$), constants ($a_1, \ldots, a_n, \ldots$), predicate and function symbols of all adicities ($F_1^1, \ldots, F_n^1, \ldots, F_1^2, \ldots, F_n^2, \ldots$), the same countable and expressively adequate set of truth-functional connectives as PL_ω, the existential and universal quantifiers and the identity predicate $=$, standard formation rules, and its standard consequence relation $\vDash_{\mathsf{FOL}_\omega}$.[11] $\mathit{Sen}(\mathsf{FOL}_\omega)$ is FOL_ω's set of sentences. We sometimes write $\langle \mathit{Sen}(\mathsf{FOL}_\omega), \vDash_{\mathsf{FOL}_\omega} \rangle$ for FOL_ω, to emphasize that FOL_ω consists of a set of sentences equipped with a consequence relation; similarly, we may write $\langle \mathit{Sen}(\mathsf{PL}_\omega), \vDash_{\mathsf{PL}_\omega} \rangle$ for PL_ω.

The third node of our triangle is natural language, here represented by English (understood to include technical outposts such as mathematical, scientific, and legal language, and not limited to the homely vocabulary from which the Bruin and Zeno arguments are drawn). In line with virtually all linguists and philosophers, we assume

[11] Models of FOL_ω are thus assumed to have non-empty domains.

that the set $Sen(E)$ of English sentences is countably infinite. The reason is that the lexicon of English is finite and its formation rules allow for sentences of arbitrary finite length.[12]

Now, we know that over a finite domain an existential claim is materially equivalent to a finite disjunction of less complex formulas; likewise, a universal claim is materially equivalent to a conjunction. In a domain of two things denoted by constants a and b, respectively, $\exists xFx$ is materially equivalent to the disjunction $Fa \vee Fb$, or, if we formalize the atomic sentences Fa and Fb as p and q, respectively, to $p \vee q$. More generally, if the elements of the domain are $a_1, \ldots, a_n$, $\exists x\phi(x)$ is materially equivalent to $\phi(a_1) \vee \cdots \vee \phi(a_n)$ and $\forall x\phi(x)$ to $\phi(a_1) \wedge \cdots \wedge \phi(a_n)$; similarly for predicates of adicity greater than 1. Repeatedly applying this procedure, we can turn any first-order sentence into a Boolean (truth-functional) combination of atomic sentences of the form $F_i a_j$ (or $R_i a_{j_1} \ldots a_{j_k}$ more generally), and we may uniformly replace each such sentence with a distinct sentence letter. This is a cheap way of ensuring *material* equivalence of interpreted propositional and first-order formalizations *over a particular finite domain*. However, this "local" approach has two limitations. First, $Fa \vee Fb$ is only materially equivalent to $\exists xFx$ over that specific two-membered domain and interpretation; furthermore, we wish to allow for infinite domains, where the simple trick just mentioned will not do, since PL_ω does not admit infinitely long sentences. Clearly, then, a more sophisticated approach is needed.

To that end, suppose $\mathcal{L}_1$ and $\mathcal{L}_2$ are two logics, with respective sets of sentences $Sen(\mathcal{L}_1)$, $Sen(\mathcal{L}_2)$, and respective consequence relations $\vDash_{\mathcal{L}_1}, \vDash_{\mathcal{L}_2}$. We write $\mathcal{L}_i$ for $\langle Sen(\mathcal{L}_i), \vDash_{\mathcal{L}_i}\rangle$, where $i = 1, 2$. The map $j : Sen(\mathcal{L}_1) \to Sen(\mathcal{L}_2)$ is a *conservative translation* (or a *faithful translation*) just when, for all $\Gamma \subseteq Sen(\mathcal{L}_1)$ and $\delta \in Sen(\mathcal{L}_1)$,

$$\Gamma \vDash_{\mathcal{L}_1} \delta \text{ if and only if } j(\Gamma) \vDash_{\mathcal{L}_2} j(\delta)$$

A *bijective conservative translation* is a conservative translation $j : Sen(\mathcal{L}_1) \to Sen(\mathcal{L}_2)$ that is also a bijection. Since two logics related by a bijective conservative translation are the same up to the relabeling of sentences, I shall call a bijective conservative translation a *consequence isomorphism* and two logics related in this way *consequence isomorphic*.[13]

[12] If $Sen(E)$ is assumed to be finite, the propositionalist's task is even easier.

[13] This agrees with the terminology in "Capturing Consequence." There, I also introduced the term "consequence homorphism," but now prefer the more common "conservative translation." Usually, "translation" relates to deductive consequence, but here we apply it to semantic consequence.

Following standard terminology, any map Φ from the set of English sentences to $Sen(\mathcal{L}_i)$ is a *formalization.*

One of the main facts proved in "Capturing Consequence" is that there is a consequence isomorphism j from FOL_ω to PL_ω, or from $\langle Sen(\mathsf{FOL}_\omega), \vDash_{\mathsf{FOL}_\omega}\rangle$ to $\langle Sen(\mathsf{PL}_\omega), \vDash_{\mathsf{PL}_\omega}\rangle$, to be more precise. As it turns out, and unbeknownst to me at the time of that article's publication, a version of this result had already appeared in section 2 of Jeřábek's paper, "The Ubiquity of Conservative Translations."[14] Jeřábek gives a syntactic proof, whereas my proof was semantic. I will not repeat either argument here but simply mention three facts that my proof exploited and which convey the gist of the argument. The first is that any two countable atomless Boolean algebras are isomorphic. As a result, FOL_ω's Lindenbaum algebra is isomorphic to PL_ω's Lindenbaum algebra.[15] The second is that FOL_ω and PL_ω are both compact. Intuitively, this means that consequence facts in each logic are determined by facts about their Lindenbaum algebras. The third fact is that both $Sen(\mathsf{FOL}_\omega)$ and $Sen(\mathsf{PL}_\omega)$ are countably infinite. Hence each equivalence class in $Sen(\mathsf{FOL}_\omega)$ under $\vDash_{\mathsf{FOL}_\omega}$-equivalence is countably infinite; similarly for $Sen(\mathsf{PL}_\omega)$'s $\vDash_{\mathsf{PL}_\omega}$-equivalence classes.

It follows from the existence of the consequence isomorphism j from FOL_ω to PL_ω that for any map Φ_1 from the set $Sen(E)$ of English sentences to $Sen(\mathsf{FOL}_\omega)$ there is a map Φ_2 from $Sen(E)$ to PL_ω such that the formalization of an English argument is deemed valid by formalization Φ_1 iff it is deemed valid by formalization Φ_2. The propositional formalization Φ_2 thus does no worse (and no better) than the first-order formalization Φ_1 at matching what we may call

[14] I am grateful to a referee for this JOURNAL for drawing my attention to Diego Pinheiro Fernandes, "Translations: Generalizing Relative Expressiveness between Logics," unpublished manuscript (2018), arXiv:1706.08481v1, which in turn led me to Emil Jeřábek, "The Ubiquity of Conservative Translations," *The Review of Symbolic Logic*, v (2012): 666–78. The present article's principal aim is to draw some philosophical consequences from the result.

[15] The elements of $\langle Sen(\mathsf{PL}_\omega), \vDash_{\mathsf{PL}_\omega}\rangle$'s Lindenbaum algebra consist of the equivalence classes of PL_ω-sentences quotiented by $\vDash_{\mathsf{PL}_\omega}$-equivalence. Two elements $[\gamma_1]$ and $[\gamma_2]$ of this Boolean algebra are equal if and only if $\vDash_{\mathsf{PL}_\omega} \gamma_1 \leftrightarrow \gamma_2$. Join, meet, and complement are defined in ${}^{\mathsf{PL}_\omega}/_{\vDash_{\mathsf{PL}_\omega}}$ as usual:

$$[\gamma_1] \vee [\gamma_2] = [\gamma_1 \vee \gamma_2]$$
$$[\gamma_1] \wedge [\gamma_2] = [\gamma_1 \wedge \gamma_2]$$
$$\overline{[\gamma]} = [\neg\gamma]$$

where we are using the symbols $\wedge$, $\vee$ ambiguously. ${}^{\mathsf{FOL}_\omega}/_{\vDash_{\mathsf{FOL}_\omega}}$, the Boolean algebra of FOL_ω quotiented by $\vDash_{\mathsf{FOL}_\omega}$-equivalence, is similarly defined. For more elementary facts about Lindenbaum algebras, see, for example, Peter Hinman, *Fundamentals of Mathematical Logic* (Wellesley, MA: A. K. Peters, 2005), pp. 74–79.

English consequence. Put less formally, propositional formalizations are no worse, as a class, than first-order ones at capturing English consequence.[16]

The above answers the second objection in section I: whatever first-order formalization Φ_1 you choose, there is a propositional formalization Φ_2 that does just as well as Φ_1 at capturing validity/invalidity facts. Propositional logic is *not* inferior to predicate logic as far as respecting the validity/invalidity of natural-language arguments goes.

III. THE GRAMMATICAL CHALLENGE

Moving beyond "Capturing Consequence" now, note that the consequence isomorphism from FOL_ω to PL_ω encountered in the previous section need not respect a sentence's grammatical form. Consider the following three English sentences:

(A) Ann is tall and Bob is tall.
(B) Ann is tall.
(C) Bob is tall.

The implicational relations that hold among these sentences may be summarized by saying that A implies each of B and C, and that B and C jointly imply A, but that B does not imply A, C does not imply A, B does not imply C, and nor does C imply B.[17]

One way to capture these implicational facts in PL_ω is to formalize A as $p \wedge q$, B as p, and C as q.[18] This formalization intuitively respects the English sentences' grammatical form—or at least the form that propositional logic can discern. But plenty of other formalizations will do the trick. We might, for example, formalize A as $\neg p$, B as $p \to (q \wedge r)$, and C as $p \to (\neg q \wedge r)$. As is easily verified, the resulting

[16] What exactly is English consequence? For our purposes, it can be anything you like. Note that logical *pluralists*, as opposed to *monists*, think there are various consequence relations (note the plural) we can read into English. Each of these results from looking at English through a particular theoretical lens, and none of them is any better than the others. Readers sympathetic to logical pluralism should take English consequence to be consequence understood from one of the various theoretical perspectives they believe is correct. In the framework of Jc Beall and Greg Restall, *Logical Pluralism* (Oxford: Oxford University Press, 2006), this could be the perspective of a classical logician (who takes logical relations among interpreted sentences to be classically governed), *or* an intuitionistic logician, *or* a relevance logician, *or* someone who takes consequence to be necessary truth-preservation. See Paseau, "Capturing Consequence," *op. cit.*, for more on this point.

[17] The full set of implicational facts involving these three sentences contains many others deducible from this description and obvious facts about implication; for example, A implies A follows from the fact that any argument in which the conclusion is one of the premises is valid.

[18] To put it in terms of formalization functions, we let $\Phi : Sen(E) \to Sen(\mathsf{PL}_\omega)$ be defined by $\Phi(A) = p \wedge q$, $\Phi(B) = p$, $\Phi(C) = q$; the other values of Φ are immaterial.

propositional sentences stand in the same implicational relations to one another as A, B, and C. The propositional-consequence structure of each of the two triples $\langle p \wedge q, p, q \rangle$ and $\langle \neg p, p \rightarrow (q \wedge r), p \rightarrow (\neg q \wedge r) \rangle$ replicates that of $\langle A, B, C \rangle$: the first element in the triple implies each of the other two, the second and third jointly imply the first but do not do so individually, the second does not imply the third, nor does the third imply the second, and so on.

Now respecting grammatical form is usually a constraint on formalization. Even if the criterion is not often made explicit, it is usually implicit. This is the reason we formalize "It is sunny" as p in propositional logic rather than, say, $p \wedge (q \vee \neg q)$; although equivalent to p, $p \wedge (q \vee \neg q)$ reads more grammatical structure into "It is sunny" than can be found there. Similarly, "There is a planet" is usually formalized as $\exists x Px$ in first-order logic rather than its equivalent $\neg \forall x \neg Px$, because $\exists x Px$ corresponds more closely, grammatically, to "There is a planet" than $\neg \forall x \neg Px$ does.

How to spell out the grammatical constraint on formalization is tricky. That the evaluation of grammatical form is in the eye of the beholder only serves to heighten the difficulty. A minimalist syntactician will, for example, take the syntax of a sentence such as "There is a planet" to be a particular labeled phrase structure, following the program set out by, say, Radford;[19] a bare phrase structure theorist[20] will use unlabeled tree diagrams; a third might do it differently. Turning to logicians, a propositionalist logician sees only propositional structure, whereas a first-orderist sees only first-order-formalizable structure. In particular, propositionalists and first-orderists disagree over the grammatical form a formalization must respect, since their notions of grammatical form are informed by their logic. First-orderists cannot well complain that propositionalists have failed to respect the grammar of the sentence "All women are mortal" by not formalizing it as a sentence with a leading universal quantifier. For propositionalists recognize no such form.[21]

We appear to have reached a dialectical impasse. By the result cited in the previous section and proved in "Capturing Consequence," first-orderists cannot object to propositionalism on the grounds that it fails to capture the logical structure of language as accurately as first-order

[19] Andrew Radford, *Minimalist Syntax: Exploring the Structure of English* (Cambridge, UK: Cambridge University Press, 2004), inspired by Noam Chomsky, *The Minimalist Program* (Cambridge, MA: MIT Press, 1995).

[20] See Chomsky, *The Minimalist Program, op. cit.*

[21] See Alex Oliver, "A Few More Remarks on Logical Form," *Proceedings of the Aristotelian Society*, XCIX (1999): 247–72, which nicely illustrates the moral that there is no logic-neutral grammatical standpoint to take when discussing logical form.

logic does. For propositional logic can match first-order logic like-for-like in this respect. And, as it has now emerged, nor can first-orderists object that propositionalists fail to reflect formal structure beyond the propositional. For propositionalists will retort that there is no such thing. That, after all, is precisely what is at stake between propositionalists and their opponents. A standoff seems inevitable.

As an aside, it is worth clarifying that "form" here means formal structure as captured by a formal language. Contrast and compare the arguments "Bruin is a bear, therefore there is a bear" and "Bruin is a bear, therefore there is a dog." A propositionalist can recognize that the first argument is valid and the second invalid because the predicates in each argument's conclusion are different, and crucially, "bear" appears in both the premise and the conclusion of the first argument but appears in the premise and not the conclusion of the second. Just like anybody else, propositionalists recognize that the validity of an English argument can turn on the predicates it features. What the propositionalist denies, however, is that this difference must be captured in a *formal* language that contains predicates. For as she sees it, the complex grammatical apparatus of English can, for the purposes of capturing validity, be reduced to sentential form. (Compare the first-orderist, whose formalizations also greatly simplify the grammar of English in other ways.)

Returning to our main thread, we appeared to face a dialectical standoff: propositionalists and first-orderists will interpret the grammatical constraint in their own favored way, so that from their perspective they themselves respect it and their opponent violates it. And yet, first appearances notwithstanding, there *is* a way to make some progress. For since first-order logic contains the propositional connectives, it too can discern propositional structure. So the first-orderist can insist that propositional formalizations should reflect the propositional structure she—the first-orderist—discerns. Take the sentence "Bruin is a bear and Fido is a dog," which the first-orderist formalizes as the conjunction $Bb \wedge Df$. For the first-orderist, the propositional form of the sentence "Bruin is a bear and Fido is a dog" is $\Phi_1 \wedge \Phi_2$ where Φ_1 and Φ_2 are propositionally atomic. As she sees it, the propositionalist would be *mistaken by her own lights* if she assigned the English sentence a structure other than $\Phi_1 \wedge \Phi_2$. Propositionalists, a first-orderist can reason, must at least respect sentences' propositional form. Wittgenstein praised Russell for showing that surface form was distinct from logical form;[22] but whatever its relation to surface form, propositional logical form as the first-orderist conceives it

[22] Wittgenstein, *Tractatus*, *op. cit.*, 4.0031.

must be respected by the propositionalist. So a grammatical constraint that the first-orderist can reasonably impose on the propositionalist is that the latter's formalizations should propositionally match the first-orderist's.

There is a strong reading of this constraint, which the propositionalist cannot but reject. Namely: where the first-orderist discerns no propositional structure, the propositionalist should follow suit. In other words, clauses that are formalized as propositionally atomic by the first-orderist should also be formalized as propositionally atomic by the propositionalist. The propositionalist should reject this strong reading because it rules out propositionalism from the get-go. The premise and conclusion of the Bruin argument ("Bruin is a bear, therefore there is a bear") are, for instance, first-order-formalized as Bb and $\exists xBx$, both of which are propositionally atomic; to insist that the propositionalist must formalize them as atomic sentences is to rule out propositionalism from the outset. For the propositionalist would, in that case, be left with two options: either formalize the premise and conclusion as the same sentence letter (p), from which it would follow that the argument with the premise and conclusion switched round—"There is a bear, therefore Bruin is a bear"—is valid, which it is not; alternatively, formalize them as distinct sentence letters (p and q), and thereby fail to respect the validity of the argument "Bruin is a bear, therefore there is a bear." So propositionalists will reject this strong reading out of hand.

The only grammatical constraint the first-orderist can reasonably impose on the propositionalist is this. The propositionalist's formalization must match the first-orderist's by matching any propositional complexity the first-orderist discerns. Otherwise, the first-orderist will claim that the propositionalist cannot even get the *propositional* structure of sentences right, something she is well capable of. Naturally, the propositionalist can, and in some cases will, discern *further* propositional structure than the first-orderist; but this further structure must be supplementary. To put it another way, the propositionalist's formalization of a given sentence must result from the first-orderist's by interpreting schematic formulas which for the first-orderist are propositionally atomic.

Let us put it more formally. Suppose that a first-orderist formalizes sentence s as $B(\phi_1, \ldots, \phi_n)$, where B is a Boolean operator and $\phi_1, \ldots, \phi_n$ are propositionally atomic. In this case, we call $B(\phi_1, \ldots, \phi_n)$ the sentence s's *complete propositional form.* For example, the first-orderist formalizes "Bruin is a bear and Fido is a dog" as $Bb \wedge Df = \wedge(Bb, Df)$, with Bb and Df taken to be propositionally atomic. Given the first-order formalization $B(\phi_1, \ldots, \phi_n)$, the propositionalist's formalization must also take the form $B(\psi_1, \ldots, \psi_n)$, where $\psi_1, \ldots, \psi_n$ are propositional formulas that *may* or *may not* be atomic. This, I have argued, is the only grammatical constraint that the first-orderist can reasonably impose on the propositionalist. Any other constraint the propositionalist just cannot accept.

The constraint applies only to the overall propositional form of the sentence, and not to any propositional subformulas within the first-order formalization. Take, for example, the formula $\forall x(Fx \rightarrow Gx) \vee Ha$, which first-order-formalizes "All friars are generous or Albert is holy." The sentence's propositional form that must be respected by the propositionalist is $\phi_1 \vee \phi_2$, where $\phi_1 = \forall x(Fx \rightarrow Gx)$ and $\phi_2 = Ha$. The fact that $\forall x(Fx \rightarrow Gx)$ has a non-atomic propositional subformula, namely $Fx \rightarrow Gx$, is not something the propositionalist must acknowledge in her formalization. Since propositionalists do not accept sentences with universal quantifiers as capturing the form of anything, they are not required to somehow—whatever exactly that would mean—respect the fact that, as the first-orderist sees it, ϕ_1 has an embedded conditional.

To summarize: respecting grammar is a constraint interpreted differently by different logicians. The only dialectically acceptable constraint the first-orderist can impose on the propositionalist seems to be:

> If the first-order formalization of s is $B(\phi_1, \ldots, \phi_n)$, where B is a Boolean operator and $\phi_1, \ldots, \phi_n$ are propositionally atomic, then s's propositional formalization must also have the form $B(\psi_1, \ldots, \psi_n)$, where $\psi_1, \ldots, \psi_n$ are atomic or complex propositional formulas.

IV. MATCHING CONSEQUENCE AND GRAMMAR

We may now answer both objections to propositionalism in one fell swoop by proving that propositional logic can match first-order logic both implicationally and grammatically. More precisely, there is a map $i : Sen(\mathsf{FOL}_\omega) \rightarrow Sen(\mathsf{PL}_\omega)$ such that

- i is a conservative translation;
- i respects the boxed constraint at the end of the previous section.

For the argument, let $j : Sen(\mathsf{FOL}_\omega) \rightarrow Sen(\mathsf{PL}_\omega)$ be a consequence isomorphism, whose existence was proved in "Capturing Consequence." Consider the subset PA of $Sen(\mathsf{FOL}_\omega)$ that consists of sentences that are propositionally atomic ("PA") in the sense of the previous section—in other words, first-order sentences *s* whose complete propositional form is *s*. In yet other words, these are sentences whose leading constant (propositional connective or quantifier) is not propositional, for example, a sentence such as $\forall xFx$ or $\forall x(Fx \to Gx)$; they are not sentences such as $Fa \to Ga$ or $Hb \vee Rbc$ or $Qabc \leftrightarrow \forall xFx$, whose leading constant is propositional. We let $j_{\restriction PA}$ be *j*'s restriction to *PA*. If σ is an element of $Sen(\mathsf{FOL}_\omega)$ not in *PA* then σ has the form $B(\phi_1, \ldots, \phi_n)$ for *B* a Boolean operation and $\phi_1, ..., \phi_n$ all elements of *PA*.[23] Now extend $j_{\restriction PA}$ to a map i on the whole of $Sen(\mathsf{FOL}_\omega)$ by the following stipulation:

$$i(\sigma) = \begin{cases} j_{\restriction PA}(\sigma) & \text{if } \sigma \in PA \\ B(j_{\restriction PA}(\phi_1), \ldots, j_{\restriction PA}(\phi_n)) & \text{if } \sigma \notin PA \end{cases}$$

where, in the second alternative (that is, $\sigma \notin PA$), σ's complete propositional form is $B(\phi_1, \ldots, \phi_n)$. Notice that in this case, $\phi_1, \ldots, \phi_n$ are all in *PA*, so that $B(j_{\restriction PA}(\phi_1), \ldots, j_{\restriction PA}(\phi_n)) = B(j(\phi_1), \ldots, j(\phi_n))$.

To show that i is a conservative translation, it is sufficient to show that $i(\sigma)$ is PL_ω-equivalent to $j(\sigma)$, since j is a consequence isomorphism. This is immediate for all σ in *PA*, since i is by definition identical to j on *PA*. And for σ not in *PA*, the PL_ω-equivalence of $B(j_{\restriction PA}(\phi_1), \ldots, j_{\restriction PA}(\phi_n)) = B(j(\phi_1), \ldots, j(\phi_n))$ and $j(\sigma) = j(B(\phi_1, \ldots, \phi_n))$ follows from *j*'s construction. The argument is extremely simple given the details of the proof in "Capturing Consequence" but would require reintroducing a good deal of that article's machinery, so I will merely sketch the main idea in a footnote.[24] The upshot is that $i(\sigma)$ and $j(\sigma)$ are always PL_ω-equivalent, so that i is a conservative translation.

We note in passing that though j is a consequence isomorphism, i need not be. Although i is injective on *PA*, it may map an element of $Sen(\mathsf{FOL}_\omega) \setminus PA$ and an element of *PA* to the same PL_ω-sentence; for example, it may map $\forall xFx \wedge \forall xGx$ (not in *PA*) and $\forall x(Fx \wedge Gx)$ (in *PA*) not just to equivalent propositional sentences (like j does) but to one and the same sentence (unlike j). A PL_ω-formalization obtained in this way may thus formalize two distinct English sentences, formal-

[23] If σ is in *PA* it also has this form with *B* the identity operator and $n = 1$. Observe also that any Boolean operation expressible using FOL_ω's connectives can be expressed in exactly the same way using PL_ω's connectives, since the two languages contain the same truth-functional connectives.

[24] As shown in "Capturing Consequence," j is a "lift" of the Boolean algebra isomorphism between PL_ω's Lindenbaum algebra and FOL_ω's Lindenbaum algebra; that is, an element α of $Sen(\mathsf{FOL}_\omega)$ is mapped to an element β of $Sen(\mathsf{PL}_\omega)$ only if $[\alpha]$ is mapped by the Boolean algebra isomorphism to $[\beta]$. Thus $B(j(\phi_1), \ldots, j(\phi_n))$ is logically equivalent (in PL_ω) to $j(B(\phi_1, \ldots, \phi_n))$.

ized as distinct but logically equivalent FOL_ω-sentences, as one and the same PL_ω-sentence. This in itself should not in any way count as a defect of propositional logic. We expect adequate formalizations to map different English sentences to the same formal one. For example, (standard) logic is insensitive to the distinction between active and passive voice, so that if just one of the relevant predicates is available, "Ann loves Bob" and "Bob is loved by Ann" are formalized in the same way. In their formalizations, propositionalists may similarly discriminate less finely among logically equivalent sentences than first-orderists—they may, for example, collapse the syntactic difference the first-orderist sees between $\forall xFx \wedge \forall xGx$ and $\forall x(Fx \wedge Gx)$—but they are none the worse for it.

In "Capturing Consequence," I proved a more general result for any compact logic $\mathcal{L}$ that contains the Boolean operations. Suppose Φ is a formalization of English into such a logic; that is, $\Phi : Sen(E) \to Sen(\mathcal{L})$ is a formalization function. Then there is a conservative translation $j : \Phi(Sen(E)) \to Sen(\mathsf{PL}_\omega)$, where $\Phi(Sen(E))$ is $Sen(E)$'s image under Φ. The same argument as earlier in this section can then be rerun to show that PL_ω is able to match $\mathcal{L}$ with respect to both consequence and grammar. This applies in particular to any $\mathcal{L}$ that extends PL_ω and has a sound and complete deductive procedure.

The answer to the objections to propositionalism canvassed in section I is then as follows. For any given first-order formalization there is a propositional formalization that agrees with it about the validity/invalidity of any argument *and* simultaneously satisfies the boxed constraint at the end of section III—the only grammatical constraint propositionalists will accept, as explained. Propositional logic can seemingly rebut *both* of the section I objections.

We end this section with a third motivation for re-examining propositionalism. Our introductory motivation, recall, was that it is suggested by some of the early Wittgenstein's ideas; a second was that it is widely rejected by logicians for reasons which, when you probe them, seem feeble, and which crumble on further examination (sections II–IV). Our third motivation is ontological. Take the Quinean idea that our ontology consists of the entities that must exist if the regimentations of our best scientific theories in our canonical logic are to be true.[25] For Quine, this canonical logic is predicate logic. Which entities an interpreted theory cast in this logic commits us to has a clear answer: all and only the entities in the domain. Put more loosely, it is quantifiers that carry ontological commitment. But consider what

[25] W. V. Quine, *Word and Object* (Cambridge, MA: MIT Press, 1960).

happens when we take the attractive Quinean idea that equates our ontological commitments with those incurred by the regimentations-into-the-canonical-logic of our best theories and couple it, as many do, with the un- or anti-Quinean idea that predicate logic need not play this privileged role. Which logic should? Propositionalists have a ready answer: propositional logic. *It* is the canonical logic, they maintain.[26] But the sentences of a propositional theory have no quantifiers (nor names). So how do we read off the ontological commitments from a propositional theory? Does a theory cast in propositional logic presuppose the existence of *any* objects?

Replacing predicate logic, which allows us to quantify over and denote entities, with propositional logic, which appears not to, thus seems to have drastic consequences for ontology, at least ontology pursued in a broadly Quinean spirit. ("Broadly" meaning: minus Quine's attachment to predicate logic.) Evidently, the ideas in the previous paragraph require more development and scrutiny than we can give them here. What they suggest, though, is that there is a lot at stake more broadly in getting clear about what is right or what is wrong with propositionalism.

V. PROPOSITIONALISM REASSESSED

So what *is* wrong with propositionalism? If it cannot be faulted for either failing to account for validity/invalidity facts or for the grammatical structure it reads into English sentences, *where* does it go wrong?

I would like to offer four answers to this, in fact loaded, question. First, propositionalism *may* be faulted for its implicational weakness. For it may fail to account for logically valid arguments with an infinite set of premises any of whose finite subarguments is invalid. An example might be the argument consisting of premises of the form "There are at least *n* planets" for finite *n* and conclusion "There are infinitely many planets." In contrast, a non-compact logic such as second-order logic or any infinitary logic with countably infinite conjunction can capture this argument's validity. I shall say no more about this since propositional logic is, in this regard, clearly in the same boat as any other compact logic, and because the validity of such arguments as the one just mentioned is controversial.[27] At any rate, it is clear that the preference for first-order over propositional logic cannot originate here, since first-order logic falls down in exactly the same way propositional logic does.

[26] This claim goes beyond propositionalism as defined in the introduction, but it is a very natural addition to it.

[27] See Owen Griffiths and A. C. Paseau, *One True Logic* (Oxford: Oxford University Press, forthcoming), chapter 5, for more discussion.

The second point is that the propositionalist owes us a story about *why* only the sentential connectives are logical. Suppose she has some non-question-begging reason to distinguish "and" from "because," perhaps because the extension of "and" is invariant in some fashion that the extension of "because" is not.[28] That story must rule out quantifiers as logical and demarcate the logical constants as all and only those propositional logic recognizes. The propositionalist requires, in other words, a criterion of logicality that is independently persuasive and coincides with the standard propositional logical constants. In the absence of such a criterion, there may be nothing to recommend propositionalism, even if it is invulnerable to the section I objections. That noted, propositional logic seems once more to be in the same boat as first-order logic in this regard, as there are no plausible criteria of logicality I know of that motivate all and only the constants of first-order logic (with identity).[29]

Third, to conclude that propositional logic cannot be grammatically faulted is to misinterpret the result in the previous section. What we showed there was that propositionalists' formalization is grammatically no worse, *by their own lights*, than first-orderists'. Non-propositionalists may still fault it for producing the wrong structure of a sentence, or at any rate an incomplete one. One might insist that logic should articulate grammatical structure beyond the sentential. For instance, we might insist that quantifiers should be recognized and identifiably formalized. Now, as explained, such a demand will not be dialectially persuasive to the ardent propositionalist. But being dialectically uncompelling to a philosophical opponent is not the same as being wrong, as we all know. So propositionalists' formalizations may be grammatically inadequate even if their proponents cannot be persuaded of the fact.

Naturally, if attributing grammatical structure is not an end in itself, the debate will not stop there. Suppose that one uses a logic $\mathcal{L}$ to give an account of the semantic structure of English, by inputting grammatical form into a semantics. Schematically, we have:

$$Sen(E) \xrightarrow{\Phi} Sen(\mathcal{L}) \xrightarrow{S} \mathrm{IS},$$

where Φ is a formalization function, S is a semantics, and IS stands for "interpreted sentences." Roughly, Φ produces grammatical structure

[28] For much more on invariantism, as captured by the so-called Tarski–Sher thesis or otherwise, see *ibid.*

[29] The point may be familiar enough; for its detailed justification, the reader is once more referred to *ibid.*

and S interprets it. It remains to be shown that using propositional logic for this purpose leads to an unsatisfactory theory. For remember, anything first-order logic (or any compact logic) can do in this respect, propositional logic can seemingly also do. Whatever semantics one chooses, an analogous propositional alternative exists that achieves an extensionally equivalent result. For the propositionalist may replace the chain

$$Sen(E) \xrightarrow{\Phi} Sen(\mathsf{FOL}_\omega) \xrightarrow{S} \text{IS},$$

with

$$Sen(E) \xrightarrow{j \circ \Phi} Sen(\mathsf{PL}_\omega) \xrightarrow{S \circ j^{-1}} \text{IS},$$

where $j : Sen(\mathsf{FOL}_\omega) \to Sen(\mathsf{PL}_\omega)$ is a consequence isomorphism, as in section II, and $\circ$ is functional composition. The end result is the same interpretation of English sentences, since $(S \circ j^{-1}) \circ (j \circ \Phi) = S \circ (j^{-1} \circ j) \circ \Phi = S \circ \Phi$; but now the interpretation proceeds via a propositional formalization, that is, a propositional grammar. If the role of a logic is to provide syntactic input to be fed into a semantics, it has yet to be shown that propositionalism is not up to the task.[30]

The fourth diagnosis[31] is the one I wish to rest most on, at least if the debate concerns propositional and predicate logic's relative merits. When formalizing, we care not just about mirroring implicational structure, subject to grammatical constraints, but also about how to discover it. It is easy to check that no consequence isomorphism from FOL_ω to PL_ω can be recursive, for the following reason. A classical result of Church's is that no decision procedure for first-order logical truth exists. So if a consequence isomorphism could be recursively specified, we could use it to determine whether an FOL_ω-formula maps to a tautology of PL_ω. But being a PL_ω-tautology is a decidable property, so no such isomorphism can be recursively specified. The moral: the existence of such an isomorphism is quite distinct from our knowing what it is.

Predicate logic is thus a much better method for finding out validity facts. For a large range of arguments, if we wish to determine their validity our best bet is to use a predicate formalization. We may be able to piggyback on the predicate formalization to recast the argument in

[30] A similar point may be made about the possibly non-isomorphic map i in section IV.

[31] Mooted but not fully appreciated in Paseau, "Capturing Consequence," *op. cit.*

propositional terms, in a way that agrees with the predicate formalization validity-wise. But it would be much harder, perhaps even impossible, to use only propositional logic to determine the original argument's status in the first place. This should not, of course, be taken to mean that our minds implement a predicate-logic-based system of reasoning, or some extension of it; nor, conversely, does it exclude such a possibility. It is simplistic to assume that the reasoning system(s) we generally implement is identical to the logic logicians can use to discover, via effortful formalization, a range of validity facts. Which logic, if any, we implement in which contexts is a separate question for psychology/cognitive science.[32]

Speaking of how we actually reason, it is worth distinguishing in-principle obstacles from in-practice ones. That the consequence isomorphism between PL_ω and FOL_ω is not recursive opens up an *in-principle* epistemic gap between propositional and predicate logic. For suppose our minds are computers[33] and, less controversially, that the Church–Turing thesis is true. In that case, we may be able to come up with validity-preserving predicate formalizations without ever being able to translate them into propositional ones, since no such (conservative) translation can be recursive. This is an *in-principle* limitation on the use of propositional logic to discover logical facts.

Now it is true that *monadic* first-order logic, unlike the full-blown version, *is* decidable. (Identity aside, the only predicates in monadic first-order logic are the one-placed ones.) So the same moral does not apply to this fragment of first-order logic. What is the significance of this fact? I confess that I am not entirely sure, but the following two thoughts seem to be on the right lines. First, even if a recursive consequence isomorphism between monadic predicate logic and propositional logic exists, it may be too complex or otherwise difficult to implement; in other words, there may be an *in-practice* epistemic advantage to using monadic predicate logic as opposed to propositional logic. The second point is that our discussion casts the decidability of monadic predicate logic and the undecidability of full predicate logic in a new light. It suggests that the move from monadic to dyadic predicates and beyond—Frege's great advance—is precisely the point

[32] In an accessible introduction to psychological models of reasoning, Johnson-Laird argues that reasoning is implemented by mental models rather than logical forms (that is, mental analogues of formal languages); see Philip Johnson-Laird, *How We Reason* (Oxford: Oxford University Press, 2009). The erotetic theory of Koralus is a cutting-edge account of reasoning; see Philip Koralus, *Reason and Inquiry* (Oxford: Oxford University Press, forthcoming).

[33] Not something I take a stand on here.

at which "the language of things" becomes indispensable and supersedes "the sentence language," that is, propositional logic. The *in-principle* epistemic gulf between the use of predicate and propositional logic is really owed to the jump from monadic predicate logic to its full version, and not to the jump from propositional logic to monadic predicate logic.

An analogy with the philosophy of science further illustrates the importance of epistemic factors. A scientific theory T is by definition empirically equivalent to its set of observable consequences T^{ob}. But T may still be epistemically superior to T^{ob}, on account of its explanatory superiority and because "theories" such as T^{ob} cannot be used directly for the purposes of prediction, but must piggyback on T. Under the analogy, T plays the role of first-order logic, T^{ob} of propositional logic, and empirical equivalence that of implicational equivalence (that is, underwriting the validity of the same arguments). A similar parallel can be drawn with mathematics: nominalist theories (analogous to propositional logic) may be equivalent in their claims about the concrete (analogous to implicational equivalence) to the usual mathematics-employing ones (analogous to first-order logic). Yet, as is widely recognized, the latter are epistemically superior to the former.[34]

The fourth and final point, that propositionalism's failings are *epistemic*, is to my mind the most important. Although it may fail to capture the validity of some arguments, propositional logic is no weaker in this regard than first-order logic or any of a whole host of compact logics. Although it may be hard to motivate by criteria of logicality, other popular logics such as first-order logic are in the same boat. And although propositional formalizations may strike some as unsatisfactory or incomplete, this sort of objection is less than satisfactory because it fails to move the propositionalist. But where propositional logic falls down, at least compared to first-order logic, is in its failure to give us *insight* into validity facts, which is part of what a logic should do. *That* is propositional logic's main weakness.

VI. CONCLUSION

These days, propositionalism is either immediately disowned; or, worse, deemed so implausible that it is not even discussed; or, worst of all, not even on logicians' radar. For propositional logic is regarded

[34] For the science side of the analogy, see Bas C. van Fraassen, *The Scientific Image* (Oxford: Clarendon Press, 1980). For the mathematics one, see Hartry Field, *Science without Numbers* (Princeton, NJ: Princeton University Press, 1980); and John Burgess and Gideon Rosen, *A Subject with No Object* (Oxford: Oxford University Press, 1997).

as too weak to capture the rich logical structure of language. Philosophical logic has moved far beyond even the stronger logic of the *Tractatus*. But, as I hope to have shown, simple reasons for dismissing propositionalism are, well, simple-minded. Propositional logic can mimic the consequence relation of any compact countable logic that extends it, and it does so in a way that respects the only grammatical constraint the propositionalist can agree to. In light of these two facts, propositionalism has a much stronger hand than is usually thought. The technical results outlined here seem to suggest that the predicate-versus-propositional-logic dispute results in a stalemate, provably so. It absolutely does *not* result in an emphatic victory for the former, as is conventionally thought.

Despite that, I agree with the consensus that predicate logic does after all have the upper hand. Yet my reason is different. Propositional logic's principal shortcoming is not its "weakness," as is often thought; any such weakness only shows up when it is pitted against non-compact logics such as second-order logic. Its failings are instead epistemic. Propositional logic does not enable us to discover implicational facts in the way (some) stronger logics do. The world may be a totality of Boolean compounds of elementary propositions, as the author of the *Tractatus* would have had us believe. But to discover the totality's logical structure, we must use a more complex tool.

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